Edexcel C4 June 2005 8b and 8c

Solve $\frac{dV}{dt} = 20 - kV$ given that $V = 0$ when $t = 0$.

Separating the variables gives

$$\int \frac{1}{20-kV} dV = \int dt$$

Integrating

$$-\frac{1}{k}\ln(20 - kV) = t + C$$

Using the initial condition that $V = 0$ when $t = 0$

$$C = -\frac{1}{k}\ln 20$$

$$-\frac{1}{k}\ln(20 - kV) = t - \frac{1}{k}\ln 20$$

Rearranging

$$\ln(20 - kV) = -kt + \ln 20$$

$$20 - kV = e^{-kt+\ln 20}$$

$$kV = 20 - 20e^{-kt}$$

$$V = \frac{20}{k}\left(1 - e^{-kt}\right)$$

Given that $\frac{dV}{dt} = 10$ when $t = 5$ find V when t=10.

Differentiating $V = \frac{1}{k}\left(20 - 20e^{-kt}\right)$

$$\frac{dV}{dt} = 20e^{-kt}$$

$$10 = 20e^{-5k}$$

$$k = -\frac{1}{5}\ln\frac{1}{2} = \frac{\ln 2}{5}$$

$$V = \frac{100}{\ln 2}\left(1 - e^{-\frac{\ln 2}{5}t}\right)$$

When t=10

$$V = \frac{10}{\ln 2}\left(1 - e^{-\ln 4}\right)$$

$$V = \frac{100}{\ln 2}\left(1 - \frac{1}{4}\right)$$

$$V = \frac{75}{\ln 2} \approx 108$$